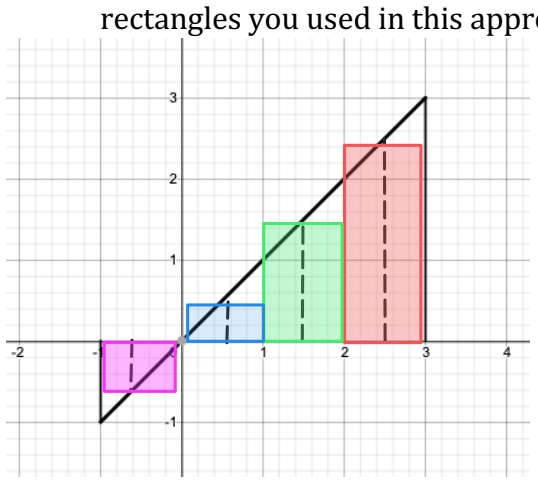
Math 5A Quiz 8 : 4.1- 4.3
20 points

On this quiz, you will evaluate $\int_{-1}^{3} x\,dx$ using the 4 methods discussed thus far:

a) Estimate the value of $\int_{-1}^{3} x\,dx$ using n= 4 subintervals and using the midpoints as sample points. Draw the rectangles you used in this approximation.



$\Delta x = \frac{b-a}{n} = \frac{3-(-1)}{4} = 1$

$(f(-\frac{1}{2}) + f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}))\Delta x$

$(-\frac{1}{2} + \frac{1}{2} + \frac{3}{2} + \frac{5}{2})\,1$

$= 4$

b) Find the exact value of $\int_{-1}^{3} x\,dx$ using the Riemann sum definition with sample points being right endpoints and the fact that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ ((See Section 4.2 Example 3)

$\Delta x = \frac{b-a}{n} = \frac{3-(-1)}{n} = \frac{4}{n}$

$x_i = a + i\Delta x$

$= -1 + i(\frac{4}{n})$

$f(x_i) = -1 + i(\frac{4}{n})$

$\int_{-1}^{3} x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

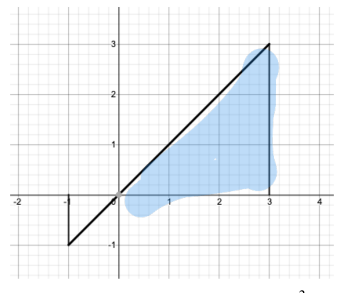
$= \lim_{n\to\infty} \sum_{i=1}^{n} (-1 + \frac{4}{n}i)\frac{4}{n} = \lim_{n\to\infty} \frac{4}{n}\sum_{i=1}^{n} (-1 + \frac{4}{n}i)$

$= \lim_{n\to\infty} \frac{4}{n}\left(-n + \frac{4}{n}\frac{n(n+1)}{2}\right) = \lim_{n\to\infty} \left(4 + \frac{8}{n^2}n(n+1)\right)$

$= \lim_{n\to\infty} \left(4 + 8 + \frac{8}{n}\right) = 4$

$\frac{8n^2+8n}{n^2}$

c) Compute $\int_{-1}^{3} x\,dx$ using the area interpretation.



$\int_{-1}^{3} x\,dx = \text{Area above} - \text{Area below} = \frac{1}{2}\cdot 3\cdot 3 - \frac{1}{2}\cdot 1\cdot 1$

$= \frac{9}{2} - \frac{1}{2} = 4$

d) Compute $\int_{-1}^{3} x\,dx$ using the second part of the Fundamental Theorem of Calculus.

$\int_{-1}^{3} x\,dx = \frac{1}{2}x^2\Big]_{-1}^{3} = \frac{1}{2}\cdot 9 - \frac{1}{2}(-1)^2 = \frac{9}{2} - \frac{1}{2} = 4$

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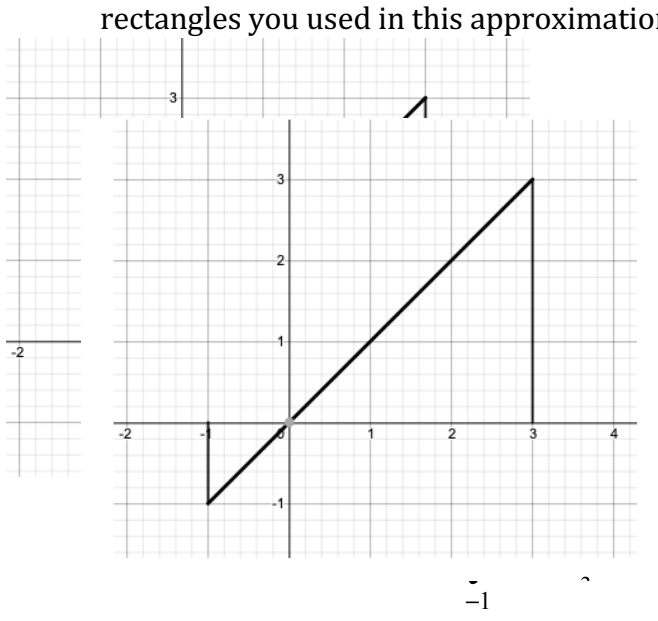

e Riemann sum definition with sample points being right endpoints and the fact that

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ ((See Section 4.2 Example 3)

c) Compute $\int_{-1}^{3} x\,dx$ using the area interpretation.

d) Compute $\int_{-1}^{3} x\,dx$ using the second part of the Fundamental Theorem of Calculus.